Fast, Flexible, and Exact Minimum Flow Decompositions via ILP

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Introduction

Background and Motivation

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Flow decomposition (FD), the problem of decomposing a **network flow** into a set of source-to-sink **paths** and associated **weights** that perfectly explain the flow values on the edges, is a classical and well-studied concept in Computer Science.

Motivation

- The main bioinformatics motivation for this paper is *multiassembly* [1], reconstruct multiple genomic sequences from mixed samples using short substrings (called *reads*) generated cheaply and accurately from next-generation sequencing technology;
- One example is the **reconstruction of RNA transcripts** from sequencing reads, which is essential to characterize gene regulation and function, development and diseases such as cancer;
- Another is the **reconstruction of viral quasispecies** which can identify different strains of a virus in a samples.

Methodology

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Definition (Flow network)

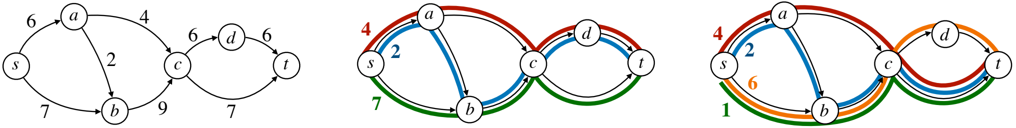
A tuple $G = (V, E, f)$ is said to be a *flow network* if (V, E) is a DAG with unique source s and unique sink t, where for every edge $(u, v) \in E$ we have an associated positive integer *flow value* f_{uv}, satisfying *conservation of flow* for every $v \in V \setminus \{s, t\}$, namely:

$$\sum_{(u,v)\in E} f_{uv} = \sum_{(v,w)\in E} f_{vw}. \tag{1}$$

Definition (k-Flow Decomposition)

A *k-flow decomposition* $(\mathcal{P}, w)$ for a flow network $G = (V, E, f)$ is a set of k s-t flow paths $\mathcal{P} = (P_1, \ldots, P_k)$ and associated weights $w = (w_1, \ldots, w_k)$, with each $w_i \in \mathbb{Z}^+$, such that for each edge $(u, v) \in E$ it holds that:

$$\sum_{\substack{i \in \{1,\ldots,k\} \text{ s.t.} \\ (u,v) \in P_i}} w_i = f_{uv}. \tag{2}$$



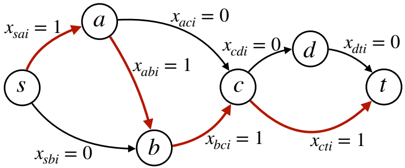
ILP Formulation

Flow Conservation

$$\sum_{(s,v)\in E} x_{svi} = 1, \qquad \forall i \in \{1,\dots,k\}, \tag{3a}$$

$$\sum_{(u,t)\in E} x_{uti} = 1, \qquad \forall i \in \{1,\dots,k\}, \tag{3b}$$

$$\sum_{(u,v)\in E} x_{uvi} - \sum_{(v,w)\in E} x_{vwi} = 0, \qquad \forall i \in \{1,\dots,k\}, \forall v \in V \setminus \{s,t\}. \tag{3c}$$



Flow Superposition

$$\sum_{i\in\{1,\dots,k\}} x_{uvi} w_i = f_{uv}, \qquad \forall (u,v) \in E. \tag{4}$$

Linearized as:

$$f_{uv} = \sum_{i\in\{1,\dots,k\}} \pi_{uvi}, \qquad \forall (u,v) \in E, \tag{5a}$$

$$\pi_{uvi} \le \overline{w} x_{uvi}, \qquad \forall (u,v) \in E, \forall i \in \{1,\dots,k\}, \tag{5b}$$

$$\pi_{uvi} \le w_i, \qquad \forall (u,v) \in E, \forall i \in \{1,\dots,k\}, \tag{5c}$$

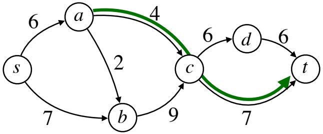
$$\pi_{uvi} \ge w_i - (1 - x_{uvi})\overline{w}, \qquad \forall (u,v) \in E, \forall i \in \{1,\dots,k\}. \tag{5d}$$

Subpath Constraints

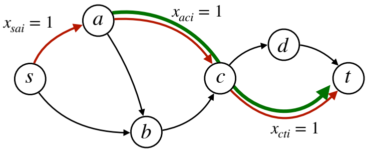
Definition (Flow decomposition with subpath constraints)

Let $G = (V, E, f)$ be a flow network. *Subpath constraints* are defined to be a set of simple paths $\mathcal{R} = \{R_1, \ldots, R_\ell\}$ in G (not necessarily s-t paths). A flow decomposition $(\mathcal{P}, w)$ *satisfies* the subpath constraints if and only if

$$\forall R_j \in \mathcal{R}, \exists P_i \in \mathcal{P} \text{ such that } R_j \text{ is a subpath of } P_i. \tag{6}$$



(a) A flow network with a single subpath constraint $R_1 = (a, c, t)$.



(b) Constraint R_1 is satisfied because for the i^{th} path we can set $r_{i1} = 1$ so that $x_{aci} + x_{cti} \geq 2r_{i1}$ holds .

Subpath Constraints - Formulation

$$\forall R_j \in \mathcal{R}, \exists P_i \in \mathcal{P} \text{ such that } R_j \text{ is a subpath of } P_i. \tag{7}$$

can be written:

$$\sum_{(u,v) \in R_j} x_{uvi} \geq |R_j| r_{ij}, \qquad \forall i \in \{1, \ldots, k\}, \forall R_j \in \mathcal{R}, \tag{8a}$$

$$\sum_{i \in \{1,\ldots,k\}} r_{ij} \geq 1, \qquad \forall R_j \in \mathcal{R}. \tag{8b}$$

Inexact Flow

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FD - ILP

Fernando H.C. Dias

Definition (Inexact flow network)

A tuple $G = (V, E, \underline{f}, \overline{f})$ is said to be an *inexact flow network* if (V, E) is a DAG with unique source s and unique sink t, where for every edge $(u, v) \in E$ we have associated two positive integer values $\underline{f_{uv}}$ and $\overline{f_{uv}}$, satisfying $\underline{f_{uv}} \leq \overline{f_{uv}}$.

Given an inexact flow network $G = (V, E, \underline{f}, \overline{f})$ the *minimum inexact flow decomposition* problem is to determine if there exists, and if so, find a minimum-size set of s-t paths $\mathcal{P} = (P_1, \ldots, P_k)$ and associated weights $w = (w_1, \ldots, w_k)$ with $w_i \in \mathbb{Z}^+$ such that for each edge $(u, v) \in E$ it holds that:

$$\underline{f_{uv}} \leq \sum_{\substack{i \in \{1,\ldots,k\} \text{ s.t.} \\ (u,v) \in P_i}} w_i \leq \overline{f_{uv}}. \quad (9)$$

Inexact Flow - Formulation

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$$\underline{f_{uv}} \leq \sum_{i \in \{1,\ldots,k\}} \pi_{uvi} \leq \overline{f_{uv}}, \qquad \forall (u,v) \in E. \tag{10a}$$

Results

Experiments Design

1. Time limit of 60 seconds;
2. Comparison of *STANDARD* with Toboggan, the implementation by [2] for their exact FPT algorithm for MFD;
3. Comparison of *SUBPATH* with Coaster, the implementation by [3] for MFDSC, which is an exact FPT algorithm extending Toboggan.
4. Comparison of *INEXACT* with IFDSolver, which is an implementation of a heuristic algorithm for MIFD by [4];
5. Three different datasets composed of RNA transcripts with range of nodes between 4 and 50;

Results

- For the *STANDARD* formulation, we could solve all instances within **20 seconds.** While the *TOBOGGAN* required at least **1 minute** for instances up to 10 flow-paths. For instances with more than 10 flow-paths, it did **not** solve within the runtime limit;
- For the *SUBPATH* version, the runtime for our formulation is below **30 seconds**, while *COASTER* cannot solve most instances;
- For the *INEXACT*, the runtime of the heuristic is **faster**, but the solution is not optimal (overestimate the number of paths by 2, in average).

Conclusions

Conclusions

- Fast: all instances were solved in under 20 seconds while state-of-the-start method requires a few minutes;
- Flexible: capable to be easily adjusted to incorporate new behaviour (as done with subpath constraints and inexact constraint) without compromising performance;
- Future Work: Cycles, improvement in inexact flow with Robust Optimization;
- `https://github.com/algbio/MFD-ILP`

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