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PHONI
Streamed Matching Statistics
with
Multi-Genome References

[accepted at DCC '21]

→ https://arxiv.org/abs/2011.05610

matching statistics (MS)

b a n d a n a a   b a n a n a a

how fits banana
into bandana?

why? MS ⇒ maximal exact matches (MEMs)
⇒ seed and extend ⇒ read alignment

matching statistics (MS)

$$\begin{array}{ccccccccc} & & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ T = & \text{b} & \text{a} & \text{n} & \text{d} & \boxed{\text{a} \quad \text{n} \quad \text{a}} \end{array}$$

$$\begin{array}{ccccccc} & & 1 & 2 & 3 & 4 & 5 & 6 \\ P = & \text{b} & \text{a} & \text{n} & \text{a} & \text{n} & \text{a} \end{array}$$
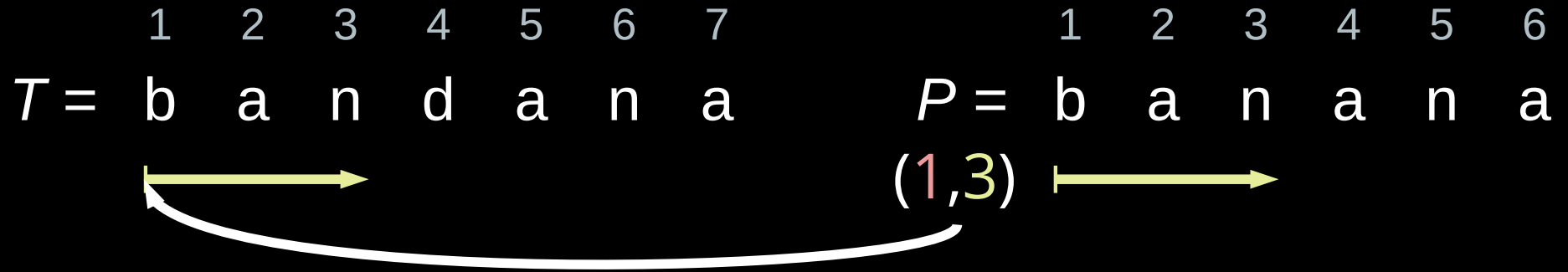
- text T
- pattern P

matching statistics (R, L) is
- $P[i .. i + L[i]\text{-}1] = T[R[i]..R[i]+L[i]\text{-}1]$
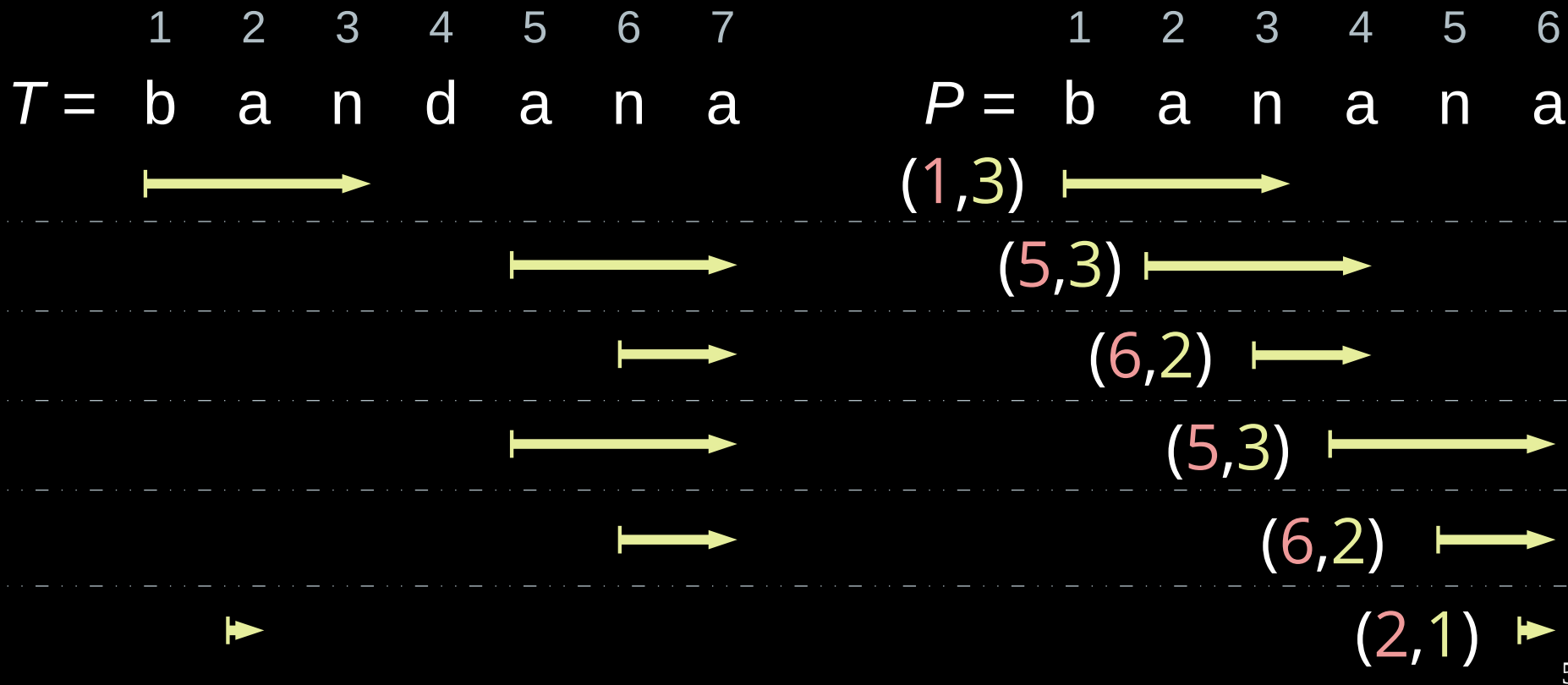- $P[i .. i + L[i]]$ does not occur

there is no $P[2..5]$ = anan in T

$$\begin{array}{ccccccc} & & 1 & 2 & 3 & 4 & 5 & 6 \\ P = & \text{b} & \boxed{\text{a}} & \text{n} & \text{a} & \text{n} & \text{a} \\ R = & 1 & 5 & 6 & 5 & 6 & 2 \\ L = & 3 & 3 & 2 & 3 & 2 & 1 \end{array}$$

matching statistics (MS)

$T =$ b a n d a n a $\quad\quad P =$ b a n a n a a

(1,3)

longest prefix of $P[1..]$ occurring in T

matching statistics (MS)

	1	2	3	4	5	6	7
T =	b	a	n	d	a	n	a

	1	2	3	4	5	6
P =	b	a	n	a	n	a

(1,3)

(5,3)

(6,2)

(5,3)

(6,2)

(2,1)

matching statistics (MS)

$T =$ b a n d a n a a

$P =$ b a n a n a

(1,3) ⊢————————▶

(5,3) ⊢————————▶

(6,2) ⊢————▶

(5,3) ⊢————————▶

(6,2) ⊢————▶

(2,1) ⊢▶

$P =$ b a n a n a
$R =$ 1 5 6 5 6 2
$L =$ 3 3 2 3 2 1

obtain MS

matching statistics (MS)

		1	2	3	4	5	6	7
T	=	b	a	n	d	a	n	a

		1	2	3	4	5	6
P	=	b	a	n	a	n	a

R not uniquely defined

(2,1)   (5,1) (7,1)

(2,1)

MS computation

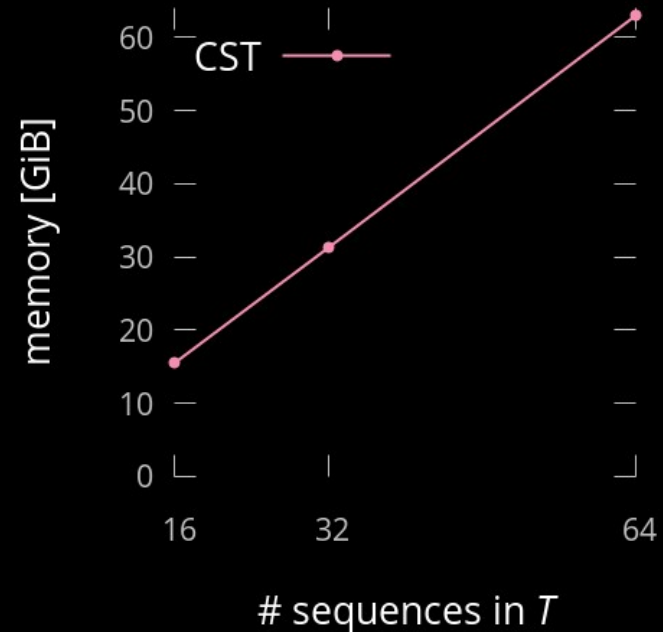| used data structure	space in bits	time		authors
		build	query	
suffix tree (ST)	$O(n \lg n)$	$O(n)$	$O(|P| \lg \sigma)$	folklore
compressed ST (CST)	$O(n \lg \sigma)$	$O(n)$	$O(|P| \lg \sigma)$	Belazzougui+ '18
r-index + grammar	$O(r \lg n + z \lg^2 n)$	$O(n \lg r)$	$O(|P| (\lg r + \lg \lg n))$	Bannai+ '20 Rossi+ '21

$n = |T|$, σ : alphabet size, r : #runs in BWT, z : #LZ77 factors

space important?

construction of CST with

- *T* : up to 1000x
  Chromosome 19 samples

- 64 GB of RAM available

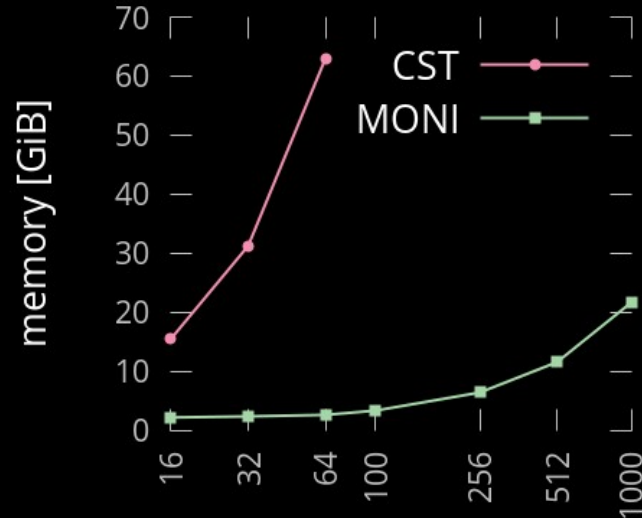⇒ can index only 64 sequences!

- Chromosome 19 needs ~ 60 MB in ASCII
- CST implementation: `cst_sct3` of sdsl-lite

space important?

MONI [Rossi+ RECOMB '21]:

- *r*-index [Gagie+ '20],

- Big BWT [Boucher+ '19],

- and data structures for MS

memory requirement scales roughly logarithmic!

sequences in *T*

log scale

MONI : augmented *r*-index

steps:

- determine *R* by backward search

- then compute *L:*

  – scan *R* and *P* from left to right

  – random access to *T* for computing *L*[i] =LCP(*T* [*R*[*i*]..], *P*[*i*..])

- needs to store *P* and *R*

- for large *P* : streaming *P* and MS becomes interesting

idea of PHONI:
compute *L* directly with
a grammar index

MS computation

BWT	F
a	$
n	a$
d	ana$
b	andana$
$	bandana$
n	dana$
a	na$
a	ndana$

for this talk simplified:

- BWT instead of *r*-index
- only compute *L*
- compute *R* with suffix array (SA)

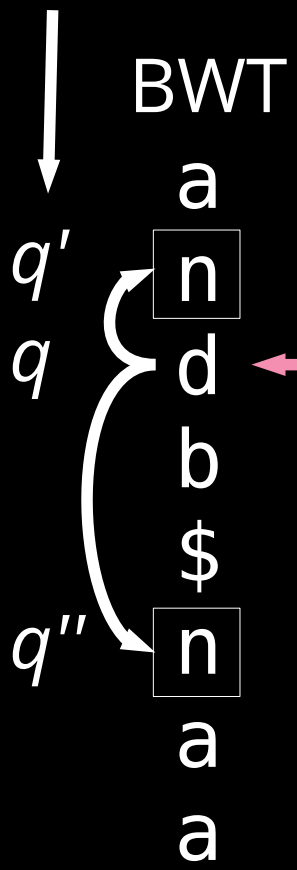(*r*-index: SA entries for each run boundary)

backward steps

BWT

F

$P =$

	1	2	3	4	5	6
	b	a	n	a	n	a

a $

n a$

d ana$

b andana$

$ bandana$

n dana$

a na$

a ndana$

L =

stop because d ≠ n

text
position

BWT

F

matching pair

i

1 2 3 4 5 6

P = b a |n| a n a

a

$

q' |n|

a$ LCP: 1

q d ← ana$

b andana$

$ bandana$

q'' |n| dana$ LCP: 0

a na$

a ndana$

continue with *q'* or *q''*:
closest neighbors of *q* in BWT
with letter n

$\quad$ LCP(*P*[*i*..], *T* [*q'*..]) and
$\quad$ LCP(*P*[*i*..], *T* [*q''*..]) :
which is longer?
$\quad\quad$ ⇒ continue with *q'*

continue backward steps

BWT

F

P =

	1	2	3	4	5	6
	b	a	n	a	n	a

a $

n   a$ LCP: 1

d   ana$

b   andana$

$ bandana$

n   dana$

a   na$

a   ndana$

L =   2   3   2   1

continue backward steps

BWT	F
a	$
n	a$
d	ana$
b	andana$
$	bandana$
n	dana$
a	na$
a	ndana$

$$P = \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ b & a & n & a & n & a \end{array}$$

$$L = \quad 2 \ 3 \ 2 \ 1$$

find continuation again

BWT

F

a
   $

n
   a$

d
   ana$

b
   andana$

$
   bandana$

n
   dana$

a
   na$

a
   ndana$

$$\begin{array}{ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ P = & b & a & n & a & n & a \end{array}$$

$$L = \quad 3 \quad 3 \quad 2 \quad 3 \quad 2 \quad 1$$

LCP: 2

we want to stream *P*, so we have not *P* for LCP queries!

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from LCP to LCE

BWT

F

a $

n a$

d ana$

d ←————————

b andana$

$ bandana$

n dana$

a na$

a ndana$

$P =$
	1	2	3	4	5	6
	b	a	n	a	n	a

$L =$ 3 3 2 3 2 1

actually: can use
previous BWT position
for LCP computation
⇒ LCE queries

LCE grammar

grammar answering longest common extension (LCE) queries

- use RePair + prefix free parsing [Gagie+ '19]

- random access on SLP [Gagie+ '20]
  SLP = straight line program (special kind of grammar)

- already used in MONI for random access on *T*

prefix free parsing (PFP)

- factorize T context-sensitively
- same substrings have nearly same factorization

$T =$

| | | S | | S | |

F_1 F_2 F_3 F_4 F_5

prefix free parsing (PFP)

- build grammar on each factor F_x independently

- build grammar on roots

$T = $

| F_1 | F_2 | F_3 | F_4 | F_5 |

LCE(p_1, p_2) = LCP(T [p_1..], T [p_2..])

- traverse from root down

- compare character-wise

$T =$

	F_1	F_2	F_3	F_4	F_5

p_1

p_2

but this is slow

time for MS per sequence

- slower than MONI

- the larger T the faster the execution of PHONI

why is the latter?

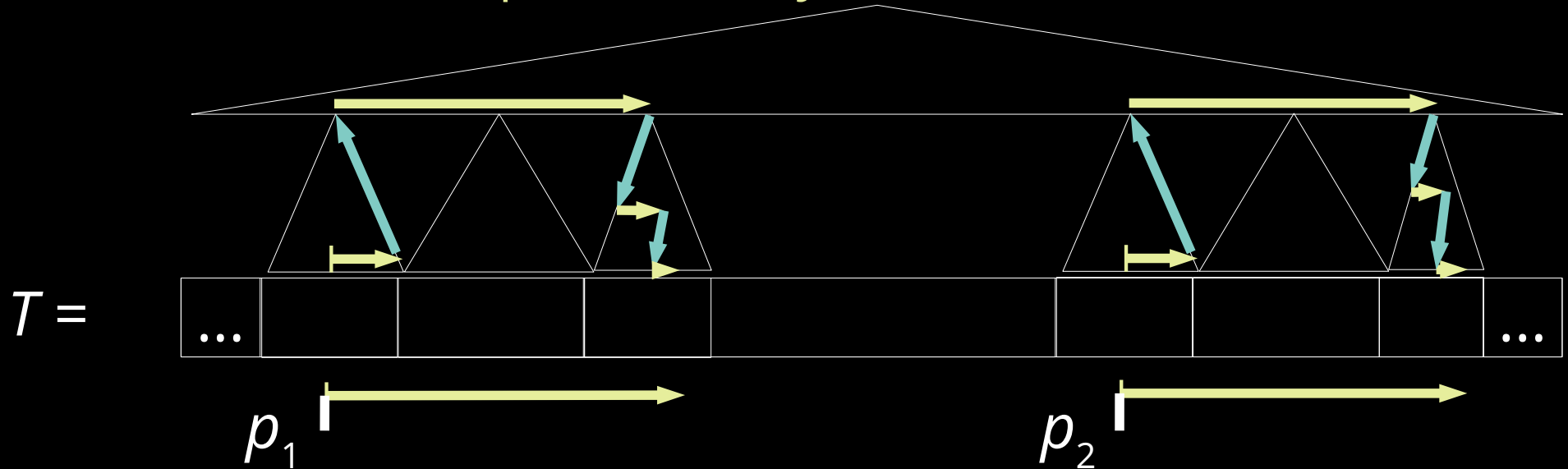- the larger T the less likely backward search fails

P = one of 10x Chromosome 19 sequences not in T

faster LCE queries

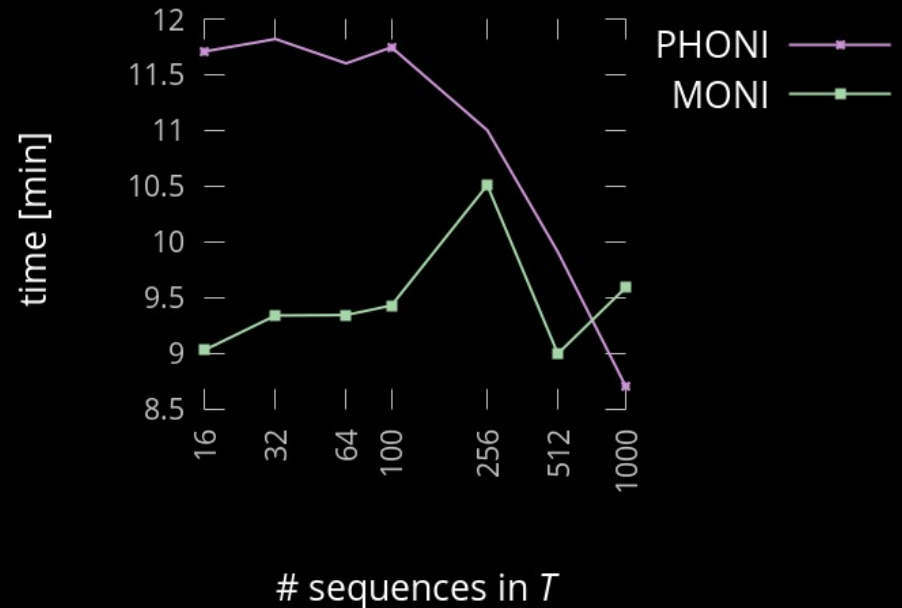- character-wise comparison will hit factor boundary at the same time
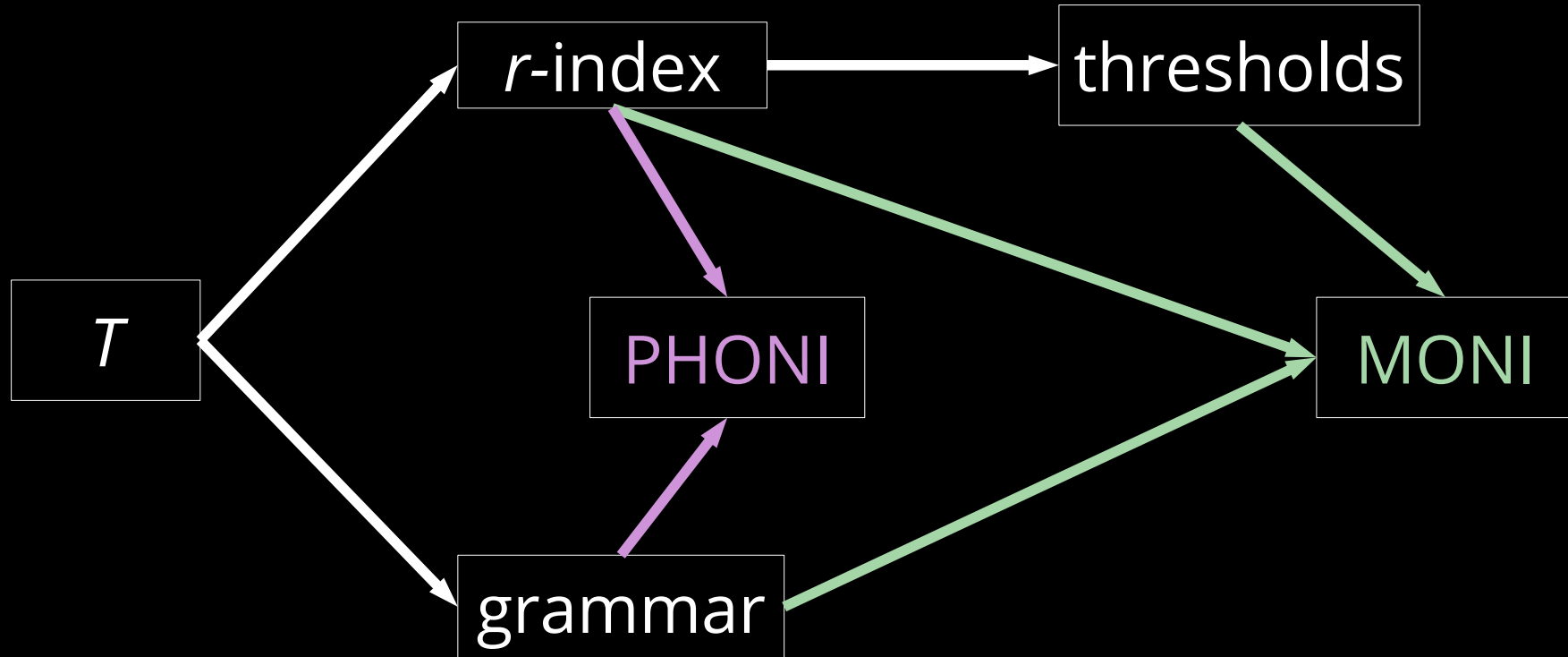
⇒ ascend and compare node by node!

$T =$ ... ...

p_1

p_2

with faster LCEs ...

PHONI faster than MONI at
T = 1000 sequences!

time for MS per sequence

(*y* axis is closer zoomed)

MONI / PHONI : build dependencies

index construction

time

total time [min]

200
180
160
140
120
100
80
60
40
20
0

16 32 64 100 256 512 1000

sequences in T

CST ●
MONI ■
PHONI ✕

space

memory [GiB]

22
20
18
16
14
12
10
8
6
4
2

16 32 64 100 256 512 1000

sequences in T

MONI ■
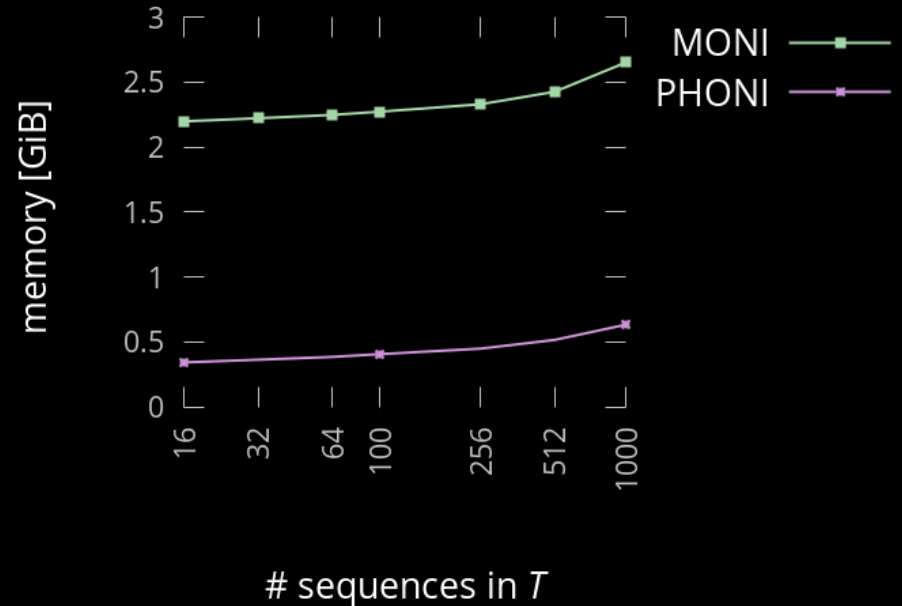PHONI ✕

gap to MONI
due to
thresholds

T consists of multiple Chromosome 19 sequences

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maximal RAM usage during queries

MONI additionally needs
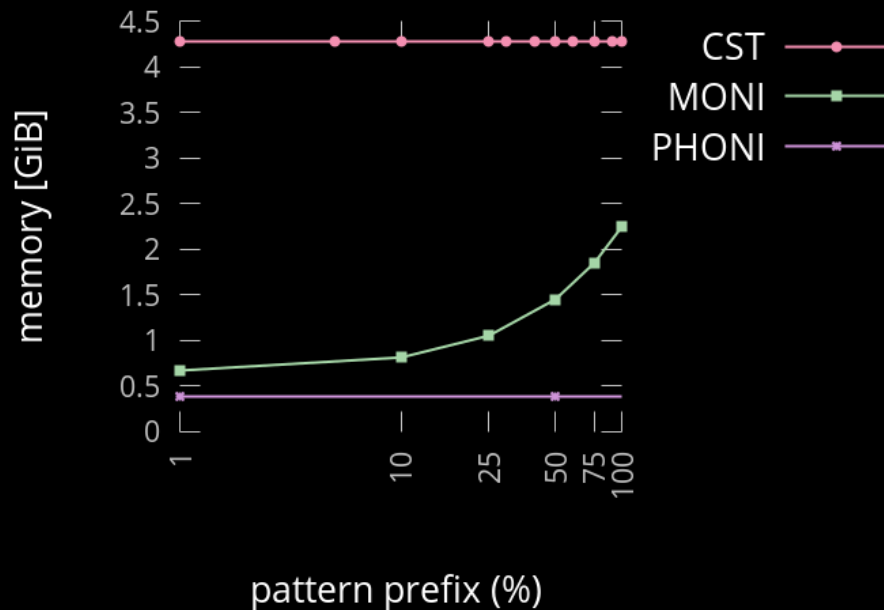
- thresholds,
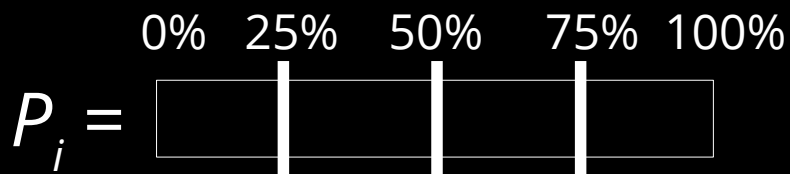
- each pattern and its R

stored in RAM

P = one of 10x Chromosome 19 sequences not in T

maximal RAM usage during queries

- fix $T = 64$ sequences

- let $P = (P_1, \ldots, P_{10})$

- compute MS for the prefix of P_i *covering x% of P_i*

$$P_i = $$

(diagram showing bar with markers at 0%, 25%, 50%, 75%, 100%)

Chart: memory [GiB] vs pattern prefix (%), with lines CST, MONI, PHONI.

what is PHONI?

- computation of matching statistics for highly repetitive T (e.g. T = pan-genome)

- stands on the shoulders of giants:
  - r-index [Gagie+ '20] [Bannai+ '20]
  - Big BWT [Boucher+ '19]
  - PFP grammar [Gagie+ '20]

our contribution:

- LCE queries on PFP grammars

- theoretically inferior to MONI, but practically competitive if
  - P is large : since we can stream P, and
  - large parts of P occur in T ⇒ only few LCE queries