Superstring Graph in compact space

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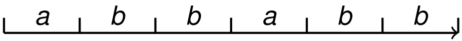
Superstring Problems


a
b
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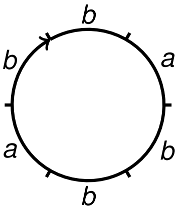
b
a
b
b
a
b

Notation

Definition [Gusfield 1997]

Let *w* a string.

- a **substring** of *w* is a string included in *w*,
- a **prefix** of *w* is a substring which begins *w* and
- a **suffix** is a substring which ends *w*.
- an **overlap** from *w* over *v* is a suffix of *w* that is also a prefix of *v*.

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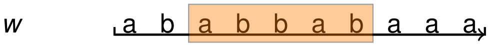

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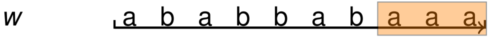

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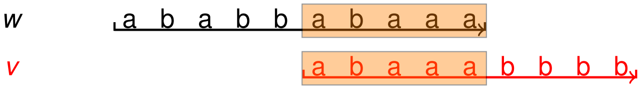

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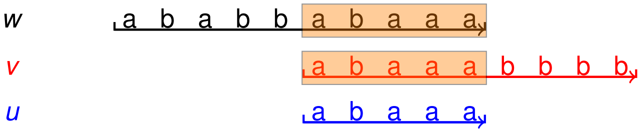

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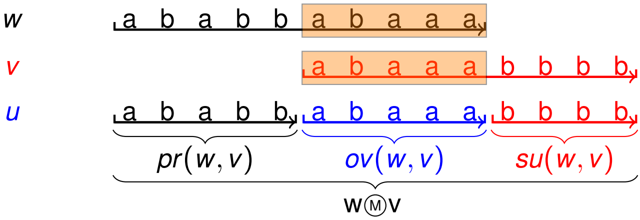
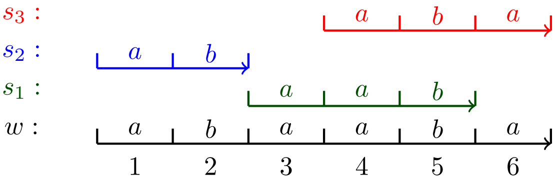
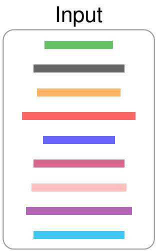


Definition

Let $P = \{s_1, s_2, \ldots, s_p\}$ be a set of strings. A *superstring* of P is a string w such that any s_i is a substring of w.



Shortest Superstrings problems



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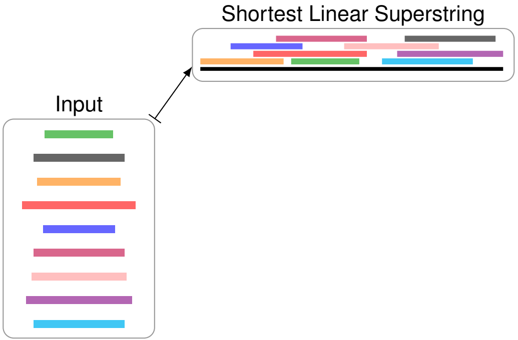

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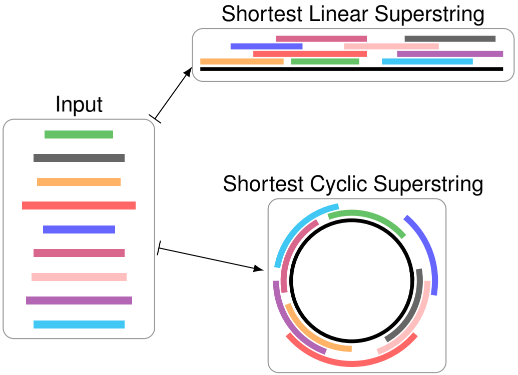

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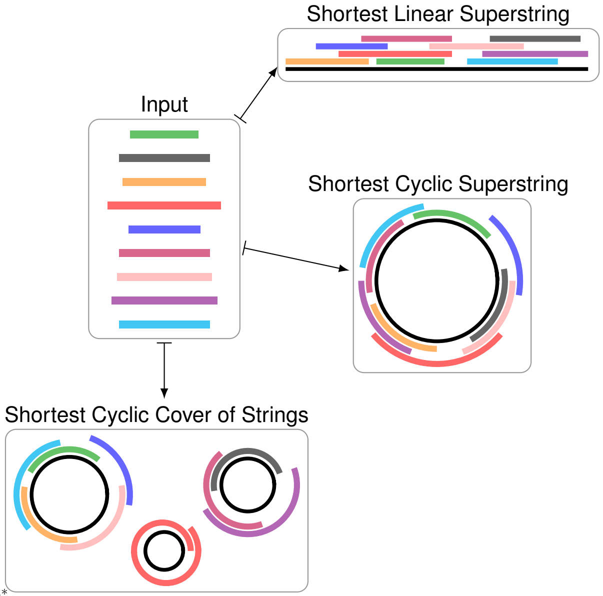
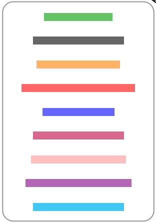


Shortest Superstrings problems

- NP-hard [Gallant 1980]
- APX-hard [Blum 1991]
- Approximation $2 + \frac{11}{30}$ [Paluch 2015]

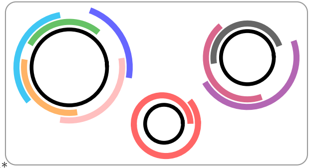
Input



Shortest Cyclic Superstring

- NP-hard [Cazaux PhD 2016]

Shortest Cyclic Cover of Strings



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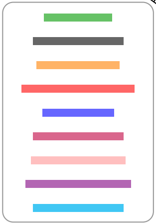

Shortest Cyclic Superstring

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Shortest Cyclic Cover of Strings

- In $O(|P|^3 + ||P||)$ time [Papadimitriou 1982]
- **Linear time in $||P||$** [Cazaux, Rivals JDA 2016]

Superstring Graph

One graph to rule them all

Greedy Algorithm for SCCS

$a\ a\ b$

$a\ b\ b\ a$

$a\ b\ a\ a$

$a\ b\ a\ b\ b$

Greedy Algorithm for SCCS

$$|ov(ababb, abba)| = |abb| = 3$$

a a b

a b b a

a b a a

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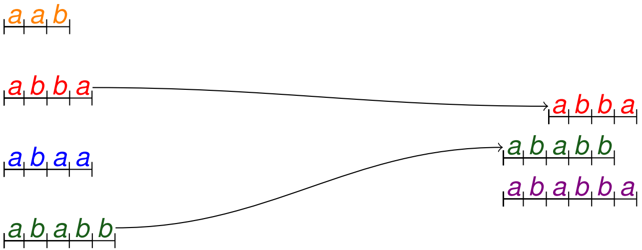

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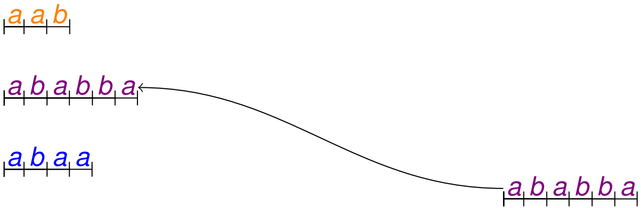
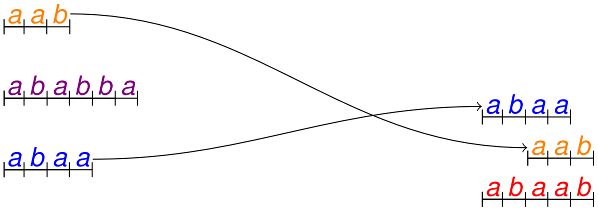


$$|ov(abaa, aab)| = |aa| = 2$$



Greedy Algorithm for SCCS

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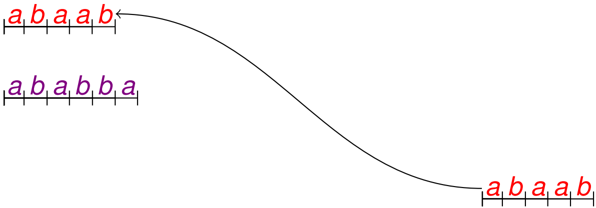

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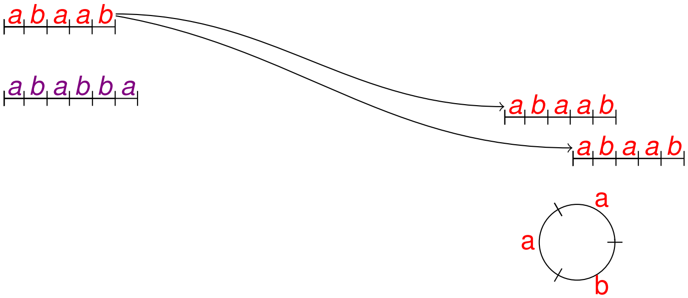

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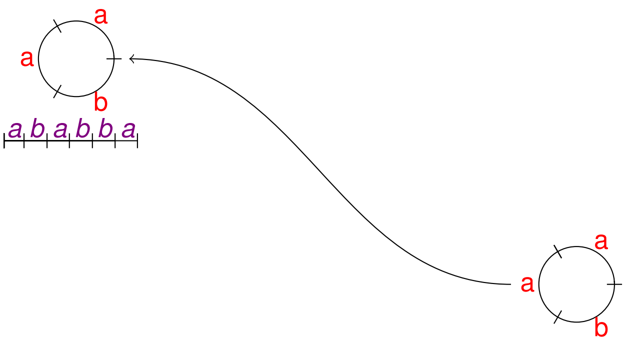
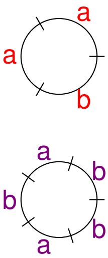
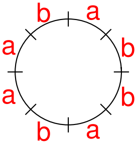


An other solution

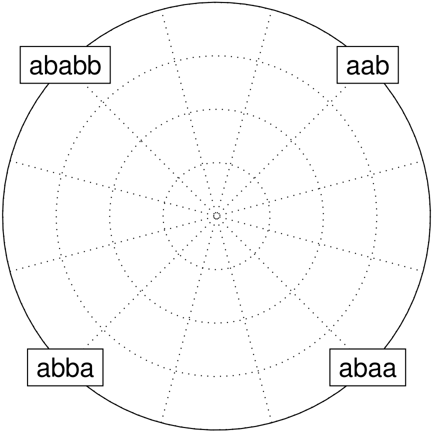




Theorem [Cazaux et al. 2014]

The greedy algorithm solves exactly the Shortest Cyclic Cover of Strings problem.

Extended Hierarchical Overlap Graph (EHOG)



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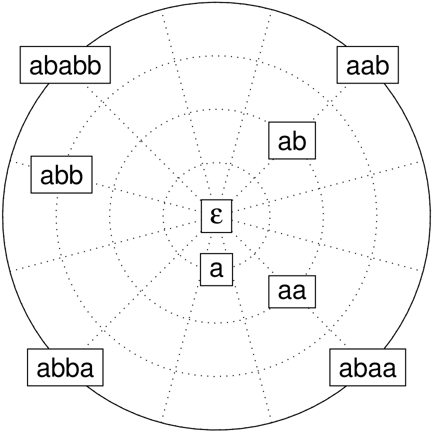

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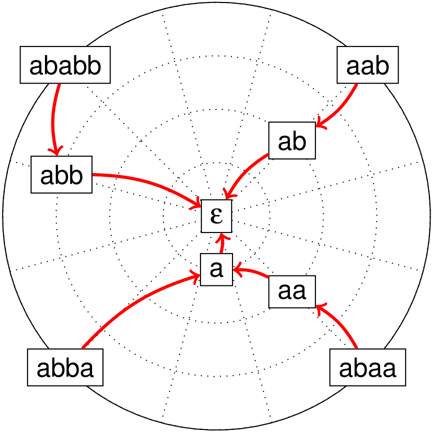

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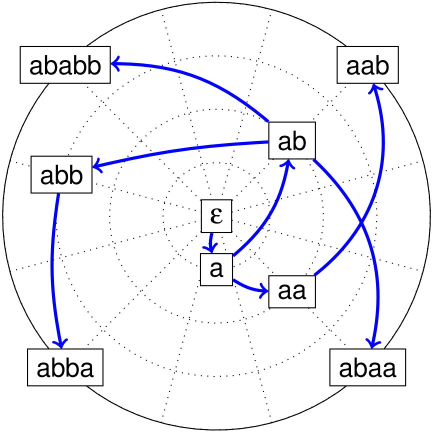

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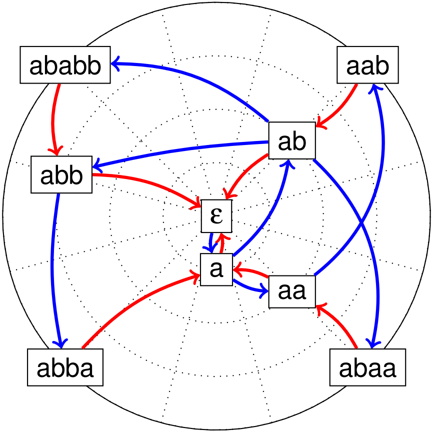

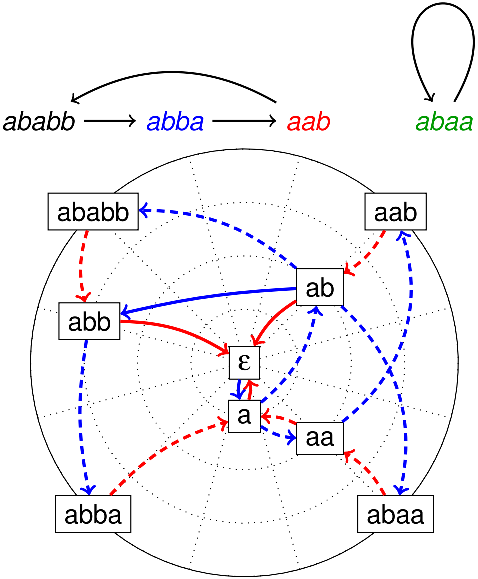
ababb
abba
aab
abaa
ababb
aab
ab
abb
ε
a
aa
abba
abaa

Definition

All the solutions of the greedy algorithm for SCCS give the same graph on the EHOG, and it is called Superstring Graph.

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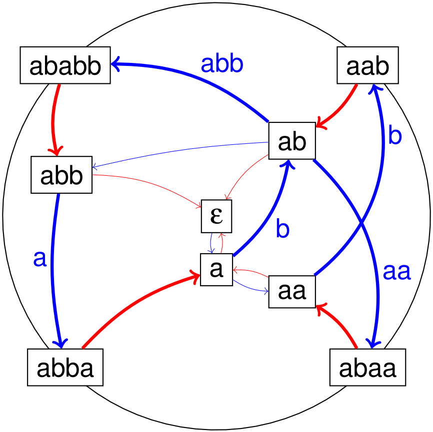
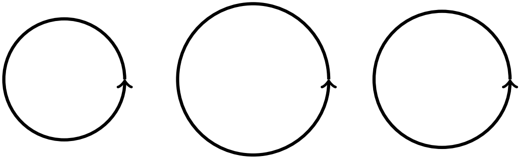
Propositions [Cazaux et al. 2015]

- The size of the Superstring Graph is linear in the size of the input.
- We can build the Superstring Graph in liner time in the size of the input.
- A labeled eulerian cycle of the Superstring Graph is a solution of the greedy algorithm for the Shortest Cyclic Cover of Strings problem.


ababb
abb
aab
b
ab
abb
ε
a
b
a
aa
aa
abba
abaa

Why do we want to compute the Superstring Graph?

Application 1: Compute bounds for optimal solution of SLS



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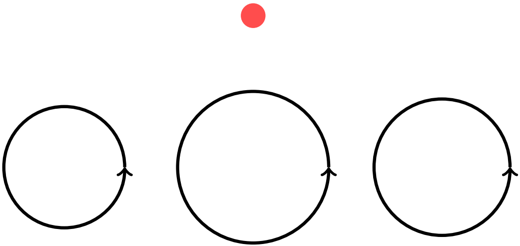

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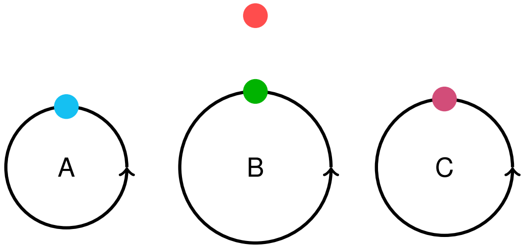

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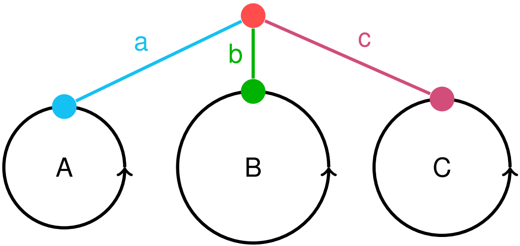

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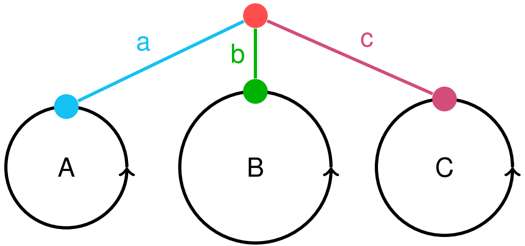
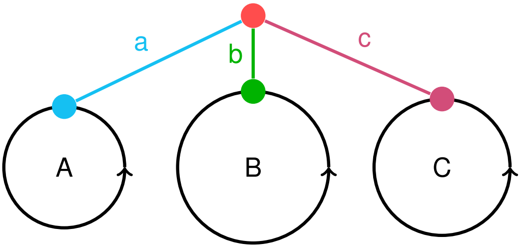


With a $\geq$ b and a $\geq$ c,

A + a + B + C $\leq$ | Optimal solution of SLS | $\leq$ A + a + B + b + C + c



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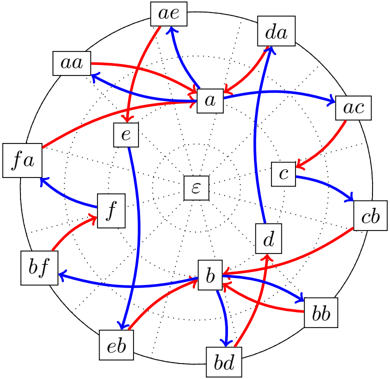
Results on real data [Cazaux et al. 2018]

Input: *E. coli* genome of 50x:
4 503 422 reads (454 845 622 symbols)

Result: length of optimal solutions
between 187 250 434 and 187 250 672
A difference of 710 symbols (**0,00038**%)

Application 2: Use SG as a genome assembly graph



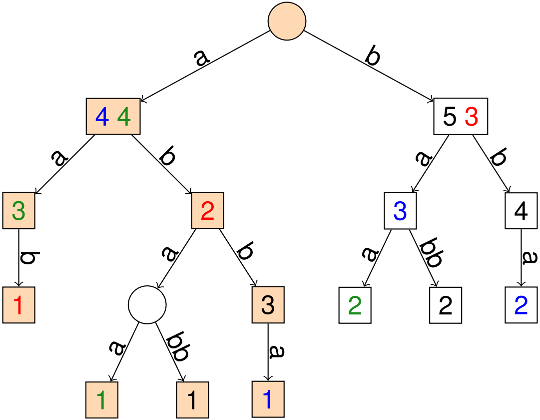
Results [Cazaux et al. 2016]

We can build a mixed cover that includes unitigs of the dBG (or Variable order dBG) in time in $O(||P||)$, and in linear space in the size of the de Bruijn Graph.

How can we **store** the Superstring Graph in compact space?

Difficulty

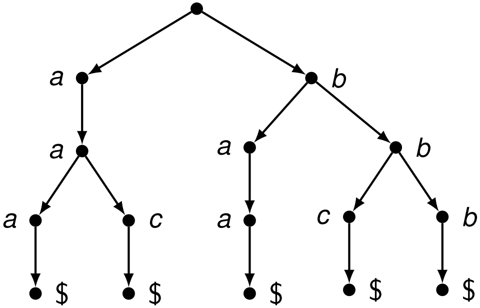
The Superstring Graph is a graph with integer value (each value in $\log ||P||$ for a set of strings P)



Propositions

Each node of *EHOG*(P) is an explicit node of the Suffix Tree of P.

Use the BWT-AC decomposition



$$S = \{aaa\$, aac\$, baa\$, bbc\$, bbb\$\}$$

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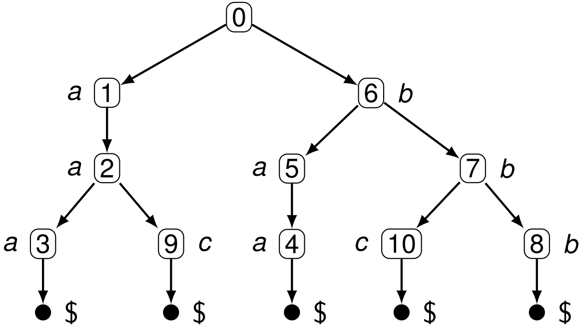

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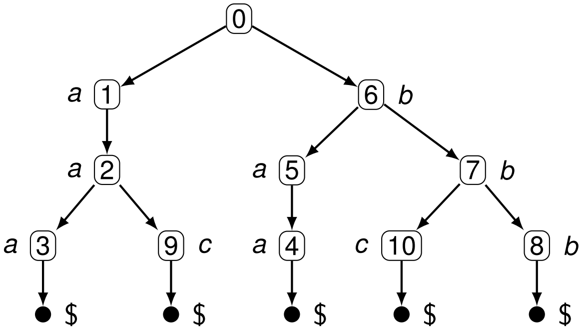

$$S = \{aaa\$, aac\$, baa\$, bbc\$, bbb\$\}$$

$$m_P = aaa\$caa\$aab\$bbb\$cbb\$$$

	0	1	2	3	4	5	6	7	8	9	10
$BWT(m_P)$	b a b a b	a a	c a	$	$	a	b a b	c b	$	$	$

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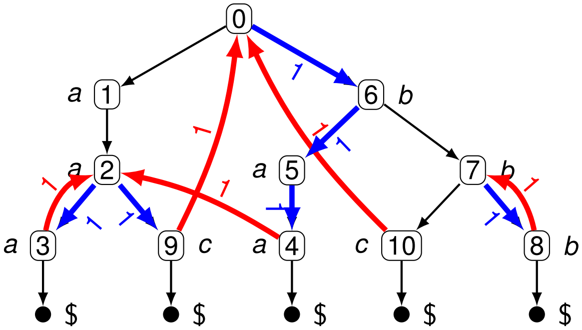

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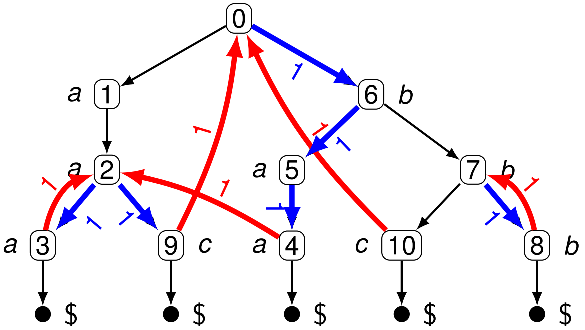

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	0	1	2	3	4	5	6	7	8	9	10
$BWT(m_P)$	*b a b a b*	*a a*	*c a*	$	$	*a*	*b a b*	*c b*	$	$	$
$Blue(m_P)$	1	1	1	01	01	01	01	1	01	01	1
$Red(m_P)$	1	1	1	01	01	1	1	1	01	01	01

Result

For a set of strings P,the size of the tables *Blue* et *Red* are bounded by $||P||$ $(=\sum_{s\in P}|s|)$.

$\rightarrow$ By using BWT and BP, we can store the Superstring Graph in $O(n\log\sigma)$ bits.

How can we **build** the Superstring Graph in compact space?

Difficulty

If we want just to compute the tables *Blue* and *Red*.

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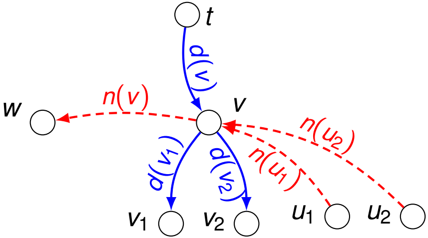

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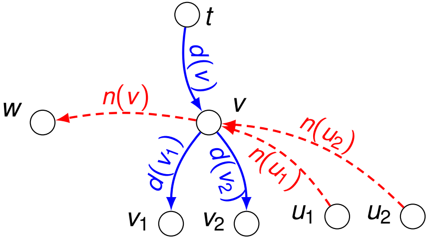

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Can we find an algorithm in $O(||P||)$ linear time and in $O(||P||)$ bits space?

Thank you for your attention